

HJ-1478

M.Sc. (Previous) Examination, 2017

MATHEMATICS

Paper - IV

Complex Analysis

Time Allowed : Three Hours

Maximum Marks : 100

Note : Attempt any two parts from each question. All questions carry equal marks.

Unit-I

- Q. 1. (a) State and prove Cauchy-Goursat theorem.
- (b) State and prove Morera's theorem.
- (c) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

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P.T.O.

(2)

Unit-II

- Q. 2. (a) Evaluate the residues of $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at the poles $z = 1, 2, 3$.
- (b) Show that linear transformation maps circle into circle.
- (c) State and prove Riemann mapping theorem.

Unit-III

- Q. 3. (a) For $z \neq 0, -1, \dots$, prove that ;

$$\bar{z} = \lim_{n \rightarrow \infty} \frac{\ln n^z}{z(z+1)\dots(z+n)}$$

- (b) State and prove Runge's theorem.

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(c) State and prove Monodromy theorem.

Unit-IV

Q. 4. (a) State and prove Jensen's theorem.

(b) Find the order of following function $\cos z$.

(c) Use Hadamard's factorisation theorem to

show :

$$\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2} \right)$$

Unit-V

Q. 5. (a) State and prove Little Picard theorem.

(b) State and prove Schottky's theorem.

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(c) Use Bloch's theorem to prove that

$$B \geq \frac{1}{72}$$

