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E-5311

M.Sc. (Ist Semester) Examination, 2021

MATHEMATICS

Paper - I

(Advanced Abstract Algebra - I)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 24

Note : Section A is compulsory and attempt any five questions from section B and any five questions from section C and any three questions from section D.

SECTION - A

Q. 1. Choose the correct answer : **5×2=10**

(i) A group G for which $Z_m(G) = G$ for some positive integer m then :

(a) G is solvable

(b) G is nilpotent

(c) G is both solvable and nilpotent

(d) None is correct

(ii) Left R module will be same as right R module if :

(a) R is a ring with unity

(b) R is a commutative ring

(c) R is without zero divisors

(d) None is correct

(iii) The characteristic and minimal polynomial for a linear operator T have :

(a) Different roots

(b) Same roots

(c) Some times different and sometimes same

(d) None of these

(iv) The index of the Nilpotent matrix

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ -2 & 1 & 1 \end{bmatrix} \text{ is :}$$

(a) 1

(b) 2

(c) 3

(d) None of these

(v) Let A be an m × n matrix over a principal ideal domain, then which of the following is correct :

(a) Row rank A = column rank B

(b) Row rank A < column rank B

(c) Row rank A > column rank A

(d) None of these

SECTION - B

Note : Attempt any five questions. **5×2=10**

Q. 1. Define commutator subgroup of a group.

Q. 2. Define free module.

Q. 3. State Schure's lemma.

Q. 4. Define characteristic roots of a matrix.

Q. 5. Define Nilpotent transformation.

Q. 6. Define invariant factors of a matrix A.

Q. 7. Define companion matrix.

SECTION - C

Note : Attempt any five questions. **5×4=20**

Q. 1. Prove that any finite p-group is solvable.

Q. 2. Prove that the minimal polynomial of a linear oprator $T \in A(V)$ divides its characteristic polynomial.

Q. 3. Find the Smith normal form of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$$

Q. 4. Let A and B be R-submodules of R-modules M and N respectively then prove :

$$\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$$

Q. 5. Prove that every Nilpotent group is solvable.

Q. 6. Let R be a ring with unity. Then prove an R module M is cyclic iff $M \cong \frac{R}{I}$ for some left ideal I of R.

Q. 7. Show that the Abelian group generated by x_1 and x_2 subject to $2x_1 = 0$, $3x_2 = 0$ is isomorphic to $Z/(6)$.

SECTION - D

Note : Attempt any three questions. **3×10=30**

Q. 1. Prove that a Group G is solvable if and only if $G^{(k)} = (e)$ for some integer R.

Q. 2. Prove that a Jordan block J may be written as the sum of a scalar matrix and a Nilpotent.

Q. 3. State and prove fundamental structure theorem for finitely generated module over a principal ideal domain.

Q. 4. Let M be a free R module with a Basis $\{e_1, e_2, \ldots\ldots e_n\}$ then prove $M \simeq R^n$.