J-1046

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - III

(Topology)

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) Prove that inter-section of two topologies is also topology. But union of two topologies is not necessary a topology.

(b) Let A be subset of a topological space, then :

$$\overline{A} = A \cup A' \left(A' = \text{Derived set} \right)$$

Q. 2. (a) Every second countable space is first countable.

(b) A metric space is separable if and only if it is second countable.

Q. 3. (a) A mapping f from a space X into another space Y is continuous if and only if :

$$f\left(\overline{A}\right) \subset \overline{f\left(A\right)} \text{ for every } A \subset X$$

(b) State and prove Tietze-extension theorem.

Q. 4. (a) Every compact Hausdorff space is a T_3-space.

(b) Every regular Lindelof space is normal.

Q. 5. (a) State and prove Urysohn's lemma.

(b) A continuous image of a compact set is compact.

Q. 6. (a) Define Hausdorff space. Prove that every compact subset of A of a Hausdorff space X is closed.

(b) Define compactness. Prove that every compact space has Bolzano-Weierstrass property.

Q. 7. (a) Define connected space. Prove that the closure of connected set is connected.

(b) The projection functions are open.

Q. 8. State and prove Urysohn's metrization theorem.

Q. 9. (a) The product space $X_1 \times X_2$ is connected iff both X_1 and X_2 are connected.

(b) Define compactness. Prove that a subset of R^n is closed and bounded iff it is compact.

Q. 10. (a) Define Net. Prove that a topological space $(x, \mathfrak{J})$ is Hausdorff iff every net in X can converge to at most one point.

(b) Define filters. Prove that a filter F on set X is an ultrafilter iff F contains all those subset of X which intersect every member of F.