

**J-1051**

**M.A./M.Sc. (Final) Examination, 2021**

**MATHEMATICS**

**Paper - I**

**(Integration Theory & Functional Analysis)**

*Time Allowed : Three Hours*

*Maximum Marks : 100*

*Minimum Pass Marks : 36*

**Note :** Attempt any five questions. All questions carry

equal marks.

**Q. 1.** State and prove that Radon-Nikodym theorem.

**Q. 2.** Prove that a union of any countable collective of positive subsets of  $X$  is positive.

**Q. 3.** State and prove that Lebesgue decomposition theorem.

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**P.T.O.**

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**Q. 4.** State Fubini's theorem and explain why :

$$\int_0^1 \left\{ \int_0^1 \frac{x^2 - y^2}{(x^2 + y^2)^2} dx \right\} dy \neq \int_0^1 \left\{ \int_0^1 \frac{x^2 - y^2}{(x^2 + y^2)^2} dy \right\} dx$$

**Q. 5.** Prove that in a normed linear space, every convergent sequence is a Cauchy sequence.

**Q. 6.** Let  $N$  be a normed linear space and let  $x, y \in N$ , then show that :

$$\left| \|x\| - \|y\| \right| \leq \|x - y\|$$

**Q. 7.** State and prove closed graph theorem.

**Q. 8.** Show that  $\ell_2^n$  is a Hilbert space.

**Q. 9.** Prove that if  $H$  is a Hilbert space, then  $H$  is reflexive.

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**Q. 10.** If  $T$  is an operator on a Hilbert space  $H$ , then

show that :

$$T = 0 \Leftrightarrow (Tx, y) = 0, \forall x, y$$

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