## M-5342

M.A./M.Sc. (IV ${ }^{\text {th }}$ Semester)

Examination, 2020
MATHEMATICS

## (Partial Differential Equations and Mechanics)

 Time Allowed : Three Hours
## Maximum Marks : 70

Note : Attempt all the following questions. Each question
carries 14 marks.
Q. 1. Obtain the complete integral of the differential equation :

$$
p^{3} x+q^{3} y=0
$$

OR
Solve the following PDE, $x U_{x}+y U_{y}+U U_{z}=0$ subject to the condition $U(x, y, 0)=x y$, where both $x>0$ and $y>0$, using the method of characteristics.
Q. 2. Find the bounded solution of :

$$
\frac{\partial y}{\partial x}=2 \frac{\partial y}{\partial t}+y
$$

if $y(x, 0)=6 e^{-3 x}$
OR

Find out the solution of :

$$
\frac{\partial^{2} u}{\partial t^{2}}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}
$$

by the method of separation of variables.
Q. 3. State and prove Cauchy-Kovalevskaya theorem.

OR

Show that the general solution of one dimensional
wave equation :

$$
\begin{gathered}
\frac{\partial^{2} u}{\partial t^{2}}=c^{2} \frac{\partial^{2} u}{\partial x^{2}} \\
\text { is } u=f(x-c t)+\phi(x+c t)
\end{gathered}
$$

(3)
Q.4. Show that :

$$
\begin{aligned}
& P=\frac{1}{2}\left(p^{2}+q^{2}\right), \\
& Q=\tan ^{-1}\left(\frac{q}{p}\right)
\end{aligned}
$$

is canonical.
OR
State and prove Lee Hwa Chung's theorem.
Q. 5. Find the relation between Poisson brackets and

Lagrange bracket.
OR
State and prove Jacobi theorem.

